PLASTIC DEFORMATION OF MATERIALS UNDER COMPLEX LOADING

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This paper is a continuation of [1]. As in [1], there is satisfactory agreement between the calculations and existing test data.

Suppose ε_x, ε_y, ε_{xy} and σ_x, σ_y, τ_{xy} are the components of the deformation and stress tensors in the z = const plane (we will consider the case of a plane deformation). Assuming that the plastic state of the element is determined solely by the deviators of these tensors, following [1], to determine the latter we will introduce the vector representation: The deformation and stress deviators and their increments will be represented in the form of the vectors Γ, T, $\Delta\gamma$, $\Delta\tau$ respectively, with polar coordinates $\Gamma, 2\Omega, \ldots, \Delta\tau, 2\varphi$ (Fig. 1)

$$\Gamma = \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \varepsilon_{xy}^2},\ \operatorname{tg} 2\Omega = \varepsilon_{xy}/(\varepsilon_x - \varepsilon_y),$$

$$\ldots$$

$$\Delta\tau = \sqrt{\left(\frac{\Delta\sigma_x - \Delta\sigma_y}{2}\right)^2 + (\Delta\tau_{xy})^2},\quad \operatorname{tg} 2\varphi = 2\Delta\tau_{xy}/(\Delta\sigma_x - \Delta\sigma_y).$$

The loading vector $\Delta\tau$ will be represented by the sum of the simple and orthogonal loads $\Delta\tau = \Delta\tau' + \Delta\tau''$. As in [1] we will assume that the orthogonal load $\Delta\tau''(\Delta\tau' = 0)$ causes a deformation increment $\Delta\gamma_*$, which is characterized by two quantities: the angle $2\beta_*$ between the direction of $\Delta\gamma_*$ and the vector of the principal shear Γ and the "shear modulus" μ_t with respect to the direction of $\Delta\tau''$

$$\Delta\tau'' = \Delta\tau \sin 2(\vartheta - \alpha) = \mu_t \sin 2\beta_* \Delta\gamma_* \ (\vartheta - \alpha = \varphi - \Omega). \tag{1}$$

Unlike [1] we also assume that the simple loading $\Delta\tau'(\Delta\tau'' = 0)$ also causes a deformation increment $\Delta\gamma'_*$, which is also characterized by two quantities (Fig. 2): the angle $2\gamma_*$ between the direction of Γ and the vector $\Delta\gamma'_*$, and the "shear modulus" μ_p with respect to the direction of $\Delta\tau'$, so that

$$\Delta\tau' = \Delta\tau \cos 2(\vartheta - \alpha) = \mu_p \cos 2\gamma_* \Delta\gamma'_* \tag{2}$$

(in [1] it was assumed that $\gamma_* = 0$).

The total increment of the shear will then be equal to $\Delta\Gamma = \Delta\gamma \cos 2\omega = \Delta\gamma_* \cos 2\beta_* + \Delta\gamma_* \cos 2\gamma_*$, and in the orthogonal direction $2\Gamma\Delta\Omega = \Delta\gamma \sin 2\omega = \Delta\gamma_* \sin 2\beta_* + \Delta\gamma'_* \sin 2\gamma_*$. Eliminating $\Delta\gamma_*$, $\Delta\gamma'_*$ using (1) and (2) we obtain

$$\Delta\Gamma = \Delta\tau\left[\frac{\cos 2(\vartheta - \alpha)}{\mu_p} + \frac{\operatorname{ctg} 2\beta_*}{\mu_t} \sin 2(\vartheta - \alpha)\right],$$
$$2\Gamma\Delta\Omega = \Delta\tau\left[\frac{\operatorname{tg} 2\gamma_*}{\mu_p} \cos 2(\vartheta - \alpha) + \frac{\sin 2(\vartheta - \alpha)}{\mu_t}\right]. \tag{3}$$

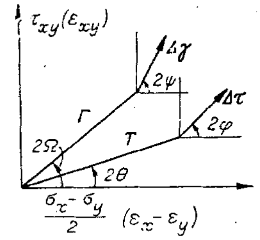

Fig. 1

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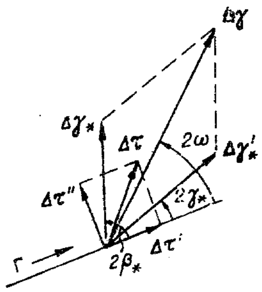

Fig. 2

We will henceforth confine ourselves to considering the simplest form of complex loading, namely, "monotonic loading," in which the main axes of the deformation tensor are constantly turned in one direction during the loading.

We will make the following assumptions: a) The quantities μ_p, μ_t, β_*, γ_* depend only on Γ in the case of monotonic loading (it is natural to assume that for a more complex deformation process these quantities will depend not only on Γ, but, possibly, on other parameters of the deformation trajectory also [1]); b) the vector of the deformation increments $\Delta\gamma$ can be represented by the sum of elastic deformation increments $\Delta\gamma_e$ and plastic deformation increments $\Delta\gamma_p$: $\Delta\Gamma = \Delta\Gamma_e + \Delta\Gamma_p$, $2\Gamma\Delta\Omega = 2\Gamma\Delta\Omega_e + 2\Gamma\Delta\Omega_p$, where $\Delta\Gamma_e = \Delta\tau \cos 2(\vartheta - \alpha)/\mu$; $2\Gamma\Delta\Omega_e = \Delta\tau \sin 2(\vartheta - \alpha)/\mu$ (μ is the elastic shear modulus).

The condition when $\Delta\Gamma_p = 0$ and $2\Gamma\Delta\Omega_p = 0$ will be taken as the condition representing the approach of complete unloading. Denoting by $2(\vartheta_u - \alpha)$ the angle which the increment vector $\Delta\tau$ makes with the direction Γ for the onset of complete unloading we obtain from (3)

$$\operatorname{tg} 2(\vartheta_u - \alpha) = -\left(\frac{1}{\mu_p} - \frac{1}{\mu}\right)\frac{\mu_t}{\operatorname{ctg} 2\beta_*}; \tag{4}$$

$$\operatorname{tg} 2\gamma_* = \left(1 - \frac{\mu_t}{\mu}\right)\left(1 - \frac{\mu_p}{\mu}\right)\operatorname{tg} 2\beta_*. \tag{5}$$

We will require that the expression $V = (1/2)\Delta\tau \cdot \Delta\gamma$ should be a "local"† potential, in other words, that the work $A = \int_{\sigma_{ij}}^{\sigma_{ij}+\Delta\sigma_{ij}} dV$, expended in the plastic deformation of the material (for a small change in the load), should not depend on the path of integration or on the order in which the load is applied. We must then have

$$\frac{\partial V}{\partial(\Delta\tau\cos 2(\vartheta - \alpha))} = \Delta\Gamma, \quad \frac{\partial V}{\partial(\Delta\tau \sin 2(\vartheta - \alpha))} = 2\Gamma\Delta\Omega.$$

Hence we also obtain from (5)

$$\frac{\operatorname{tg} 2\gamma_*}{\mu_p} = \frac{\operatorname{ctg} 2\beta_*}{\mu_t} = \pm\sqrt{\left(\frac{1}{\mu_p} - \frac{1}{\mu}\right)\left(\frac{1}{\mu_t} - \frac{1}{\mu}\right)}. \tag{6}$$

Equation (6) shows that of the four parameters β_*, γ_*, μ_p, μ_t introduced only two of them are independent (we will take these to be μ_p and μ_t).

We will now show that the model considered is a version of the theory of plastic flow. Taking (4) and (6) into account we have

$$\operatorname{tg} 2(\vartheta_u - \alpha) = -\frac{1}{\delta}, \quad \delta = \pm\sqrt{\left(\frac{1}{\mu_t} - \frac{1}{\mu}\right)\Big/\left(\frac{1}{\mu_p} - \frac{1}{\mu}\right)}; \tag{7}$$

$$\Delta\Gamma_p = \left(\frac{1}{\mu_p} - \frac{1}{\mu}\right)[\Delta\tau\cos 2(\vartheta - \alpha) + \delta\Delta\tau\sin 2(\vartheta - \alpha)], \quad 2\Gamma\Delta\Omega_p = \delta\Delta\Gamma_p, \tag{8}$$

whence it follows (since $2\Gamma\Delta\Omega_p/\Delta\Gamma_p \tan 2(\vartheta_u - \alpha) = -1$), that the plastic deformation vector $\Delta\gamma_p$ is orthogonal to the loaded surface, defined by (7), and that in the case when $\mu_t \neq \mu$ the loading surface at the point considered has an angular singularity, while the case $\mu_t = \mu$ when $\alpha \approx 0$ corresponds to the classical version of the theory of plastic flow.

† The idea of the existence of a "local" potential was suggested by E. I. Shemyakin.

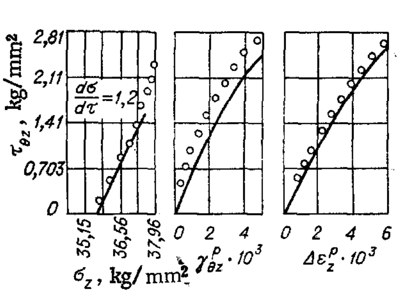

Fig. 3

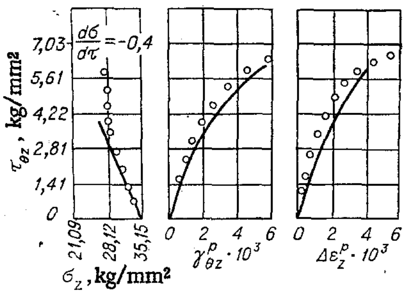

Fig. 4

As regards the choice of μ_p and μ_t we will note that μ_p is determined from the $T = T(\Gamma)$ diagram, obtained with proportional loading, while $\mu_t = \mu_t(\Gamma)$ is established from experiments on orthogonal loading. Bearing (8) in mind, we can suggest that to establish μ_p and μ_t as a function of Γ, it is sufficient to carry out only one experiment with a break in the loading trajectory at the elastic limit of this material. The calculations essentially confirm this suggestion.

Finally, we will now give the basic relationship between the increments in the stress and the increments in the deformation

$$\Delta\varepsilon_x - \Delta\varepsilon_y = \frac{\Delta\sigma_x - \Delta\sigma_y}{2\mu} + A\left[\frac{\Delta\sigma_x - \Delta\sigma_y}{2} A + \Delta\tau_{xy} B\right],$$
$$\Delta\varepsilon_{xy} = \frac{\Delta\tau_{xy}}{\mu} + B\left[\frac{\Delta\sigma_x - \Delta\sigma_y}{2} A + \Delta\tau_{xy} B\right], \quad \Delta\varepsilon_x + \Delta\varepsilon_y = \frac{\Delta\sigma_x + \Delta\sigma_y}{2k'}, \tag{9}$$

where

$$A = \sqrt{\frac{1}{\mu_p} - \frac{1}{\mu}}\cos 2\Omega \mp \sqrt{\frac{1}{\mu_t} - \frac{1}{\mu}}\sin 2\Omega; \quad B = \sqrt{\frac{1}{\mu_p} - \frac{1}{\mu}}\sin 2\Omega$$
$$\pm \sqrt{\frac{1}{\mu_t} - \frac{1}{\mu}}\cos 2\Omega; \quad k' = \text{const}.$$

The last relation expresses the law of the elastic variation of the volume for a plane deformation [1].

To compare the calculations with experimental data we will use the experimental data obtained in [2]. In those experiments, thin-walled tubular specimens made of 24S-T4 aluminum alloy ($E = 6900$ kg/mm^2 and $\mu = 2400$ kg/mm^2) were first stretched to the elastic limit so that considerable plastic deformation occurred, and were then subjected to twisting with an additional stretching load. The x axis was directed along the generatrix of the tube, and the y axis was directed along the tangential plane perpendicular to the x axis. For this loading program the principal stresses $\sigma_1, \sigma_2, \sigma_3$ are given by the following relations:

$$\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}, \quad \sigma_2 = 0, \quad \sigma_3 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2},$$

where τ_{xy} is the tangential stress, and σ_x is the stretching stress (it is assumed that a plane uniform stressed state is obtained in the specimens). In view of the fact that $\sigma_1 > 0 > \sigma_3$ (σ_1, σ_3 have different signs), then, as in the case of a plane deformation, the maximum tangential stress and the principal shear have the form

$$\tau_{max} = T = \frac{1}{2}(\sigma_1 - \sigma_3) = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}, \quad \gamma_{max} = \Gamma = \varepsilon_1 - \varepsilon_3$$
$$= \sqrt{(\varepsilon_x - \varepsilon_y)^2 + \varepsilon_{xy}^2}.$$

Further, since $\sigma_1 > \sigma_2 > \sigma_3$, this state corresponds to a state of incomplete plasticity [3, 4]. According to [3], in this case along the second principal direction a linear (quasielastic) relationship between the stresses and the deformations (or their increments) is preserved: $\Delta\varepsilon_2 = -(\nu_*/E)\times(\Delta\sigma_1 + \Delta\sigma_3)$, $\nu_* = \nu_*(\Gamma)$. Hence, from the fact that $\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 = 3K(\Delta\varepsilon_1 + \Delta\varepsilon_2 + \Delta\varepsilon_3)$ (K = const), it follows that $\Delta\varepsilon_1 + \Delta\varepsilon_3 = (\Delta\sigma_1 + \Delta\sigma_3)/2k'$ or $\Delta\varepsilon_x + \Delta\varepsilon_y = (\Delta\sigma_x + \Delta\sigma_y)/2k'$, where

$$1/2k' = 1/3K + \nu_*/E. \tag{10}$$

Repeating the discussions given above for this class of loading, we obtain the system of relations (9) in which k' is given by (10). Figures 3 and 4 show programs of certain tests and compare the results of calculation (the continuous line) with the results of experiments (we assumed in the calculations that $\mu_p = 270$ kg/mm², and $\mu_t = 960$ kg/mm²). Comparison shows that there is quite satisfactory agreement between the experimental and theoretical data.

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MOBILE LOAD ON A LAYER OF IDEALLY PACKED MATERIAL

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1. Physical Assumptions. A plane load, the shape and value of which does not change with time, moves with constant velocity U_0 over the external surface of a layer of material of constant thickness h, lying without friction on a rigid base. We will study the plane stationary motion of the medium when a shockwave $U_0 > D_0$ exists, where D_0 is the wave velocity of the corresponding pressure P_{00}, in a system of coordinate (x, y) (Fig. 1), connected with the moving load $P_0(x)$ ($P_0(x) = 0$, $x > 0$, $P_0(0) = P_{00}$). Before the wavefront the medium is unperturbed: $P = 0$, $\mathbf{U} = 0$, $\rho = \rho_0$ (P is the pressure, **U** is the mass velocity vector in the fixed system of coordinates, and ρ, ρ_0 is the current and initial density).

The material satisfies the barotropic equation of state. Its $P - \theta$ characteristic is shown in Fig. 2 (the continuous line). The equation of the straight line KM is $dP/d\rho = c^2 = \text{const}$ when $P(\theta_0) = P_{00}$ ($\theta = (\rho - \rho_0)/\rho_0$ is the volume deformation). This scheme is an idealization of the actual behavior of materials containing cavities or pores filled with easily compressed material (the dashed line in Fig. 2). The initial nonlinear part of the loading can sometimes be neglected when the characteristic pressure is higher than the pressure for which the pores collapse, and a further increment in the deformation occurs due to deformation of the matrix (for example, when the material is subjected to shock loading of considerable strength). For soft metals this region is from tens to several hundreds of kilobars. In this case the volume deformations of the matrix may remain small. For many materials the porosity is not reestablished when the load is removed, and it is possible to assume that the deformation is linear-elastic when the load is removed. Since the level of tangential stresses (determined by the relaxed amplitude of the elastic characteristic or limiting flow) is much less than the pressure of total packing, the resistance to shear can be neglected.

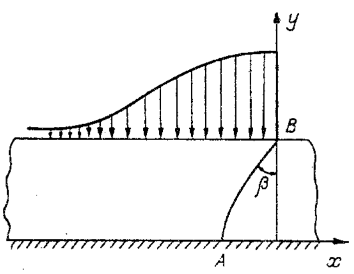

Fig. 1

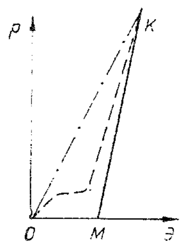

Fig. 2

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